Comparison of Elasticity, Shell Core, and Sandwich Shell Theories

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The problem of an infinite circular sandwich shell subjected to an axisymmetric radial line load is investigated using three-dimensional elasticity theory, shell core method, and sandwich shell theory due to Fulton and Schmidt. A comparison of the stresses and displacements with an exact elasticity solution is carried out for various shell parameters in order to clearly bring out the limitations of sandwich shell theories of Fulton and Schmidt as well as the shell core solution.

Nomenclature

a_i = middle surface radius of *i*th layer ($i=1,2,3$)
E_i,μ_i,G_i = modulus of elasticity, Poisson's ratio, and shear modulus for *i*th layer
$f(z)$ = axisymmetric radial line load acting on the outer surface of sandwich shell
I_0,I_1 = modified Bessel functions of first kind, zero and first order, respectively
K_0,K_1 = modified Bessel functions of second kind, zero and first order, respectively
Q = axisymmetric radial line load/unit length of circumference
r_0,r_1,r_2 = inner radius of layers 1, 2, and 3, respectively
r_3 = outer radius of layer 3
r,z = cylindrical coordinate system
t_i = (r_i-r_{i-1}), thickness of the *i*th layer
$(u)_i,(w)_i$ = radial and longitudinal displacements in the *i*th layer
$\bar{u}_1,\bar{w}_1$ = radial and longitudinal displacements at the inner interface in Eqs. (12) and (13)
u_{0i},w_{0i} = radial and longitudinal displacements at the middle surface of the *i*th layer
α = variable of integration
$(\sigma_r)_i,(\sigma_\theta)_i,(\sigma_z)_i$ = normal stresses in the *i*th layer
$(\tau_{rz})_i$ = shear stress in the *i*th layer
β = ratio of modulus of elasticity of inner/outer facing to core ($E_1/E_2=E_3/E_2=\beta$)
η_i = $(r_i-r_{i-1})/r_2$, ratio of *i*th layer thickness to radius r_2

Introduction

THE advent of high-strength, low-density composite materials has resulted in a wide use of layered and sandwich circular cylindrical shells. Analysis of sandwich shells normally is done by making use of some simplifying assumptions considering the behavior of the facings and core. It is assumed that the facings can be treated as thin shells with or without flexural rigidity while the core takes only transverse shear deformation. Reissner[1] was the first to present a method of analysis for sandwich shells assuming that the facings have no flexural rigidity and antiplane state of stress in core. Later, several investigators tried to develop sandwich shell theories and a review of this has been made by Habip.[2] For the purpose of examining the validity of sandwich shell theory, the theories developed by Fulton[3] and Schmidt[4] are considered here. Fulton[3] has derived equations which govern the behavior of elastic, unsymmetrical doubly curved shallow sandwich shells. These equations include nonlinear effects and are applicable for a sandwich shell having unequal face thicknesses and elastic properties. Schmidt[4] has derived differential equations for a sandwich shell of arbitrary shape assuming the facings to be isotropic and core orthotropic. Further, he has assumed that the facings have the same thickness and material. Recently the authors have analyzed an axisymmetrically loaded long circular cylindrical sandwich shell by the elasticity method[5] and the shell-core method.[6] In the former approach all three layers are treated as three-dimensional axisymmetrically stressed cylinders while in the latter method the core is treated as a three-dimensional axisymmetrically stressed cylinder but the facings are treated as thin shells of revolution, subjected to axisymmetric radial and shear stresses.

In this paper, first a brief description of the method of solution for a long circular cylindrical sandwich shell subjected to axisymmetric load based on elasticity theory, shell-core method, and sandwich shell theories of Fulton and Schmidt is presented. Later, a detailed comparison of results obtained from the shell-core method and sandwich shell theories is made with respect to the elasticity solution. Based on this comparison the limitations of the shell-core method and sandwich shell theories is clearly brought out.

Received March 8, 1982.

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Elasticity Solution

A sandwich cylindrical shell subjected to axisymmetric load can be analyzed as a three-layered cylinder using Love's stress function approach. For this, a stress function φ is to be selected so as to satisfy the differential equation

$$\nabla^2\nabla^2\varphi=0 \tag{1}$$

The stresses and displacements are determined from

$$\sigma_r=\frac{\partial}{\partial z}\left[\mu\nabla^2\varphi-\frac{\partial^2\varphi}{\partial r^2}\right]\quad \sigma_z=\frac{\partial}{\partial z}\left[(2-\mu)\nabla^2\varphi-\frac{\partial^2\varphi}{\partial z^2}\right]$$

$$\sigma_\theta=\frac{\partial}{\partial z}\left[\mu\nabla^2\varphi-\frac{1}{r}\frac{\partial\varphi}{\partial r}\right]\quad \tau_{rz}=\frac{\partial}{\partial r}\left[(1-\mu)\nabla^2\varphi-\frac{\partial^2\varphi}{\partial z^2}\right] \tag{2}$$

$$u=-\frac{1}{2G}\frac{\partial^2\varphi}{\partial r\partial z}\quad w=\frac{1}{2G}\left[2(1-\mu)\nabla^2\varphi-\frac{\partial^2\varphi}{\partial z^2}\right] \tag{3}$$

For a three-layered cylinder subjected to axisymmetric normal load on the outer surface (Fig. 1), the following boundary conditions can be written down.

$$\text{at } r=r_3 \quad \sigma_r=f(z); \quad \tau_{rz}=0$$
$$\text{at } r=r_0 \quad \sigma_r=0; \quad \tau_{rz}=0 \tag{4}$$

For perfect bond between the layers, the continuity conditions along a typical interface can be written as

at $r=r_{j-1}$:

$$(\sigma_r)_{j-1}=(\sigma_r)_j; \quad (\tau_{rz})_{j-1}=(\tau_{rz})_j$$
$$(u)_{j-1}=(u)_j; \quad (w)_{j-1}=(w)_j \quad (j=2,3) \tag{5}$$

The radial load acting on the outer boundary ($r=r_3$) can be expressed in terms of Fourier integral as

$$f(z)=\int_0^\infty q(\alpha)\cos\alpha z\,d\alpha$$

where

$$q(\alpha)=\frac{2}{\pi}\int_0^\infty f(z)\cos\alpha z\,dz \tag{6}$$

For the case of radial line load considered here

$$q(\alpha)=Q/\pi \tag{7}$$

The stress function φ_i for a typical ith layer ($i=1,2,3$) which satisfies Eq. (1) can be taken as

$$\varphi_i=\int_0^\infty \frac{1}{\alpha^3}[A_i(\alpha)I_0(\alpha r)+B_i(\alpha)\alpha r I_1(\alpha r)+C_i(\alpha)K_0(\alpha r)$$
$$+D_i(\alpha)\alpha r K_1(\alpha r)]\sin\alpha z\,d\alpha \quad (r_{i-1}\le r\le r_i) \tag{8}$$

where r_{i-1} and r_i are the inner and outer radius of the ith layer. The stresses and displacements can be obtained from Eqs. (2), (3), and (8). The constants $A_i(\alpha)$, $B_i(\alpha)$, $C_i(\alpha)$, and $D_i(\alpha)$ can be determined later using the boundary and continuity conditions [Eqs. (4) and (5)]. The detailed procedure and the final equations can be found in Ref. 5.

Shell-Core Method of Solution

The Kelkar and Flügge[7] technique has been extended here for a sandwich cylindrical shell, consisting of two thin facings having different thicknesses and elastic constants and a thick core, subjected to axisymmetric load. It is assumed that there is perfect bond between the core and the facing.

The analysis of core is done by using Love's stress function approach in which a stress function φ_2 having the same form as in Eq. (8) is selected. The functions $A_2(\alpha)$, $B_2(\alpha)$, $C_2(\alpha)$, and $D_2(\alpha)$ are determined from the interface continuity conditions. The stresses and displacements in the core are then determined using Eqs. (2) and (3).

The facings are considered as thin circular cylindrical shells subjected to axisymmetric radial and shear stresses. For such

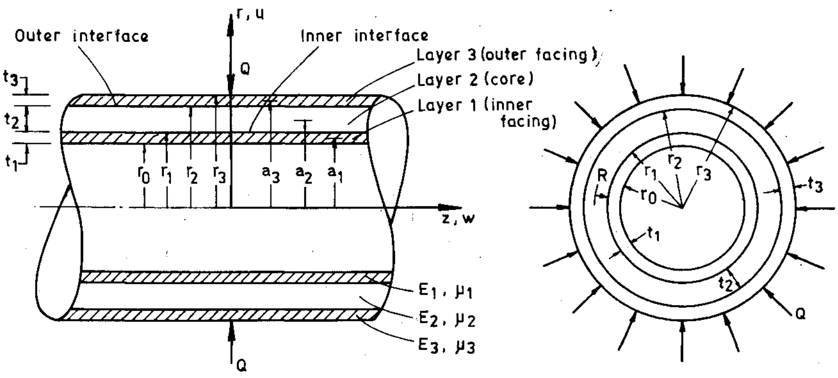

Fig. 1 Coordinate axes and dimensions.

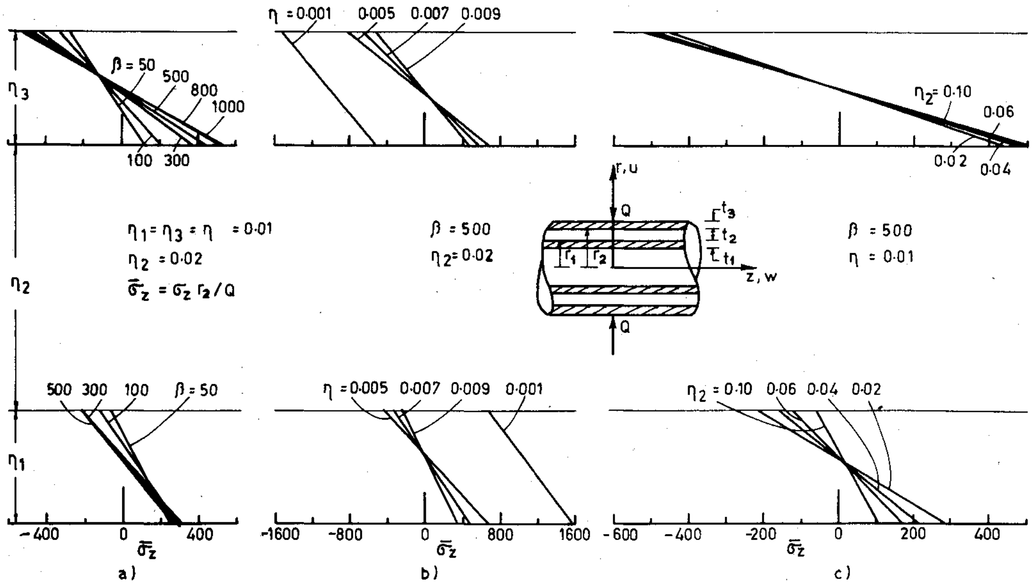

Fig. 2 Variation of longitudinal stress across the shell thickness at $z/r_2=0$ for different shell parameters (elasticity results).

a case, the governing differential equations in terms of middle plane displacements may be obtained by suitably modifying Flügge's[8] equations, as

$$a_i^2\frac{\partial^2 w_{0i}}{\partial z^2}+\mu_i a_i\frac{\partial u_{0i}}{\partial z}-\frac{t_i^2 a_i}{12}\frac{\partial^3 u_{0i}}{\partial z^3}=\pm\frac{\tau_i a_i^2}{E_i t_i}(1-\mu_i^2)$$

$$a_i\mu_i\frac{\partial w_{0i}}{\partial z}+\left(1+\frac{t_i^2}{12a_i^2}\right)u_{0i}-\frac{t_i^2 a_i}{12}\frac{\partial^3 w_{0i}}{\partial z^3}+\frac{t_i^2 a_i^2}{12}\frac{\partial^4 u_{0i}}{\partial z^4}$$

$$=\mp(\sigma_i+f(z))\frac{a_i^2}{E_i t_i}(1-\mu_i^2)\pm\frac{\partial\tau_i}{\partial z}\frac{a_i r_j}{E_i}(1-\mu_i^2) \qquad (9)$$

where σ_1, τ_1, σ_3, and τ_3 are interface radial and shear stresses, given as

$$\sigma_1=\sigma_r|_{r=r_1} \qquad \tau_1=\tau_{rz}|_{r=r_1}$$

$$\sigma_3=\sigma_r|_{r=r_2} \qquad \tau_3=\tau_{rz}|_{r=r_2} \qquad (10)$$

where σ_r and τ_{rz} are the radial and shear stresses in the core. [In Eq. (9) the upper sign corresponds to the outer facing, the lower for the inner facing, and $f(z)=0$ for inner facing. Also, $i=1$ and $j=1$ for the inner facing and $i=3$ and $j=2$ for the outer facing.]

Since the differential equations for the facings are written in terms of middle surface displacements it is necessary to relate these displacements with those at the interface. These relationships can be written as

$$\bar{u}_i=u_{0i} \qquad \bar{w}_i=w_{0i}\pm\frac{t_i}{2}\frac{\partial u_{0i}}{\partial z} \qquad (i=1,3) \qquad (11)$$

(In the preceding equation the positive sign corresponds to the outer interface and the negative sign to the inner interface.)

The continuity conditions can be written as

$$\bar{u}_1=u_2|_{r=r_1} \quad \bar{w}_1=w_2|_{r=r_1} \quad \bar{u}_3=u_2|_{r=r_2} \quad \bar{w}_3=w_2|_{r=r_2} \qquad (12)$$

Using Eq. (12) in Eq. (11) we can express u_{0i} in terms of u_2 and w_{0i} in terms of w_2 and $\partial u_2/\partial z$. Thus, the w_2, $\partial u_2/\partial z$, σ_1, τ_1, σ_3, and τ_3 expressions, obtained using Eqs. (2) and (3), are substituted in Eq. (9) to obtain four equations to solve for four unknowns: $A_2(\alpha)$, $B_2(\alpha)$, $C_2(\alpha)$, and $D_2(\alpha)$. Once these constants are obtained we can obtain stresses and displacements in the core as well as in the facings.[6]

Fulton's Method of Analysis

The differential equations in terms of displacements (neglecting higher-order terms in strain-displacement relations) for a sandwich circular cylinder shell subjected to axisymmetric load can be obtained from the general equations given by Fulton,[3] following the notations used here and taking $\mu_1=\mu_3=\mu$,

$$\frac{\mathrm{d}^3 w^*}{\mathrm{d}z^3}-\frac{\mu}{a_2}\frac{\mathrm{d}^2 u}{\mathrm{d}z^2}=0 \qquad \frac{\mathrm{d}^2 u}{\mathrm{d}z^2}=\phi-K\frac{\mathrm{d}^2\phi}{\mathrm{d}z^2}$$

$$(\bar{D}_1+\bar{D}_3)\frac{\mathrm{d}^4 u}{\mathrm{d}z^4}-(\bar{B}_1+\bar{B}_3)\left(\mu\frac{\mathrm{d}w^*}{\mathrm{d}z}-\frac{u}{a_2}\right)$$

$$+\frac{G_2h^2}{t_2}\left(\phi-\frac{\mathrm{d}^2 u}{\mathrm{d}z^2}\right)-f(z)=0 \qquad (13)$$

where

$$w^*=\frac{\bar{B}_1 w_{01}+\bar{B}_3 w_{03}}{\bar{B}_1+\bar{B}_3}; \quad \phi=\frac{\mathrm{d}\gamma}{\mathrm{d}z}$$

$$\gamma=\frac{w_{01}-w_{03}}{h}; \quad h=\frac{t_1+t_3}{2}+t_2; \quad k=\frac{t_2\bar{B}_1\bar{B}_3}{G_2(\bar{B}_1+\bar{B}_3)}$$

$$\bar{D}_i=\frac{E_i t_i^3}{12(1-\mu^2)}; \quad \bar{B}_i=\frac{E_i t_i}{1-\mu^2} \qquad (i=1,3)$$

It may be noted that the preceding equations are derived based on the assumption that the core is incompressible (leading to a constant radial displacement over the thickness of the core) and the longitudinal displacement varies linearly.

The solution for w^*, u, and ϕ can be taken in terms of Fourier integrals as

$$w^*=\int_0^\infty R(\alpha)\sin\alpha z\,\mathrm{d}\alpha, \quad u=\int_0^\infty S(\alpha)\cos\alpha z\,\mathrm{d}\alpha$$

$$\phi=\int_0^\infty T(\alpha)\cos\alpha z\,\mathrm{d}\alpha \qquad (14)$$

in which the functions $R(\alpha)$, $S(\alpha)$, and $T(\alpha)$ are determined so as to satisfy the differential Eq. (13). Once the displacements are known, stress resultants can be determined using the force-displacement relation given in Ref. 3 after specializing them for the axisymmetric case. The stresses can be computed from these stress resultants for comparison.

Schmidt's Method of Analysis

For the problem considered here the equilibrium equations given by Schmidt[4] can be written, taking $t_1=t_3=t$ and $\mu_1=\mu_2=\mu_3=\mu$, as

$$\frac{\partial\overset{+}{N}_z}{\partial z}=0$$

$$\frac{\partial^2\overset{+}{M}_z}{\partial z^2}+\frac{\overset{+}{N}_\theta}{a_2}+\left(t_2+\frac{t}{2}\right)\frac{\partial^2\bar{N}_z}{\partial z^2}=\frac{1}{2}f(z)$$

$$\frac{\partial^2\bar{M}_z}{\partial z^2}+\frac{\bar{N}_\theta}{a_2}-\frac{2E_2}{t_2}\bar{u}=-\frac{1}{2}f(z)$$

$$\bar{w}+\left(t_2+\frac{t}{2}\right)\frac{\partial\overset{+}{u}}{\partial z}-\frac{t_2}{2G_2}\frac{\partial^2\bar{N}_z}{\partial z^2}+\frac{t_2^3}{24E_2}\frac{\partial^3\bar{N}_z}{\partial z^3}=0 \qquad (15)$$

where

$$\overset{+}{u}=\tfrac{1}{2}(u_{01}+u_{03}); \qquad \bar{u}=\tfrac{1}{2}(u_{01}-u_{03})$$

$$\overset{+}{w}=\tfrac{1}{2}(w_{01}+w_{03}); \qquad \bar{w}=\tfrac{1}{2}(w_{01}-w_{03})$$

$$\overset{+}{N}_z=\tfrac{1}{2}(N_{z1}+N_{z3}); \qquad \bar{N}_z=\tfrac{1}{2}(N_{z1}-N_{z3})$$

$$\overset{+}{M}_z=\tfrac{1}{2}(M_{z1}+M_{z3}); \qquad \bar{M}_z=\tfrac{1}{2}(M_{z1}-M_{z3}) \text{ etc.} \qquad (16)$$

The equilibrium Eq. (15) can be expressed in terms of four displacements $\overset{+}{u}$, $\bar{u}$, $\overset{+}{w}$, and $\bar{w}$ using force-displacement relations given by Schmidt,[4] which result in four differential equations in terms of four displacements, the solution of which can be taken as follows, in terms of Fourier integrals:

$$\overset{+}{u}=\int_0^\infty L(\alpha)\cos\alpha z\,\mathrm{d}\alpha \qquad \bar{u}=\int_0^\infty M(\alpha)\cos\alpha z\,\mathrm{d}\alpha$$

$$\overset{+}{w}=\int_0^\infty N(\alpha)\sin\alpha z\,\mathrm{d}\alpha \qquad \bar{w}=\int_0^\infty P(\alpha)\sin\alpha z\,\mathrm{d}\alpha \qquad (17)$$

Once $\overset{+}{u}$, $\bar{u}$, $\overset{+}{w}$, and $\bar{w}$ are obtained we can determine middle surface displacements u_{01}, u_{03}, w_{01}, and w_{03} from Eq. (16). Using force-displacement relations given by Schmidt[4] and Eq. (16) we can determine N_{z1}, $N_{\theta 1}$, M_{z1}, $M_{\theta 1}$, etc. Thus, we can compute stresses in both the facings.

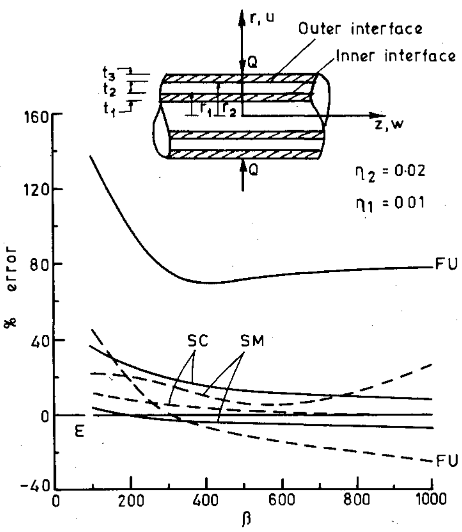

Fig. 3 Percentage error in σ_z for different values of β at $z/r_2 = 0$.

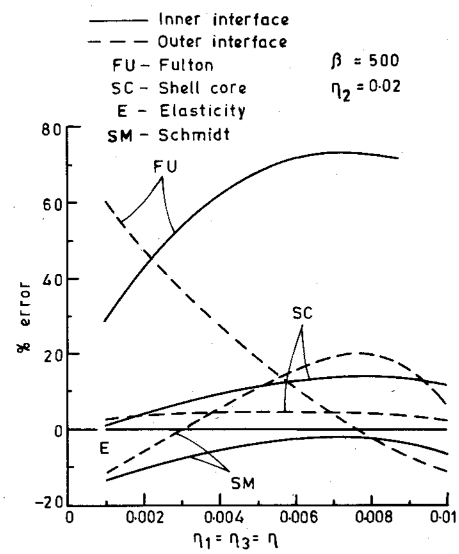

Fig. 4 Percentage error in σ_z for different values of η at $z/r_2 = 0$.

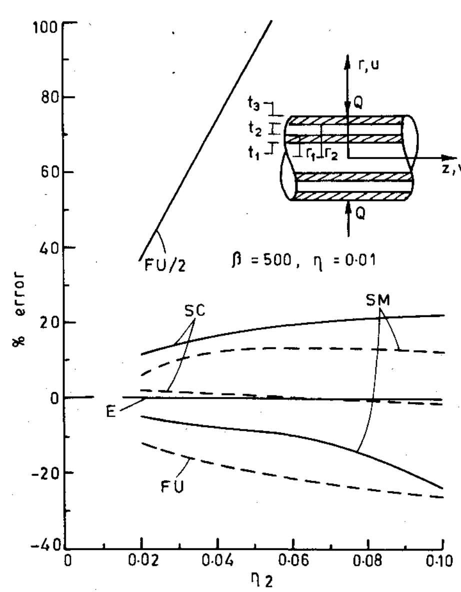

Fig. 5 Percentage error in σ_z for different values of η_2 at $z/r_2 = 0$.

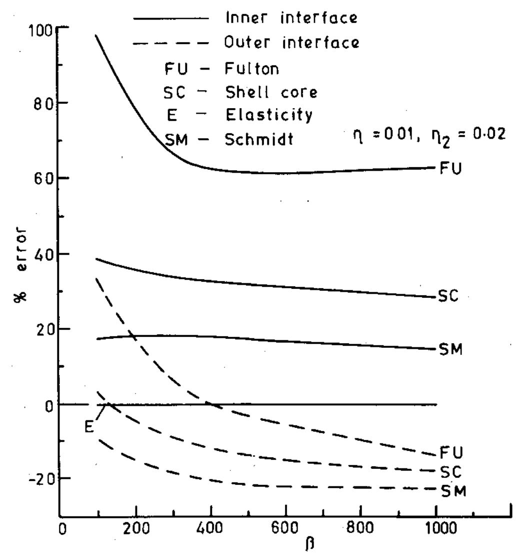

Fig. 6 Percentage error in σ_θ for different values of β at $z/r_2 = 0$.

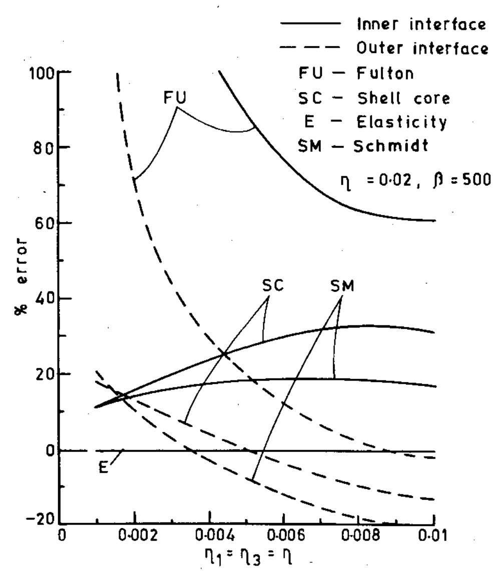

Fig. 7 Percentage error in σ_θ for different values of η at $z/r_2 = 0$.

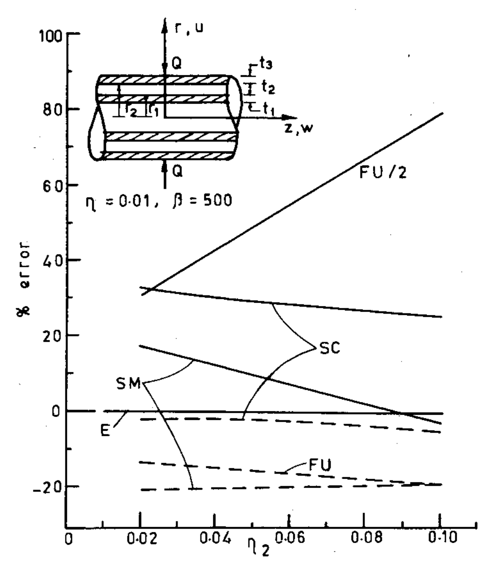

Fig. 8 Percentage error in σ_θ for different values of η_2 at $z/r_2 = 0$.

The stresses and displacements in the core can be calculated from the following equations.

$$(\tau_{rz})_2 = f_1 \qquad (\sigma_r)_2 = -R\frac{\partial f_1}{\partial z} + f_3 \tag{18}$$

and

$$(u)_2 = -\frac{1}{E_2}\left[\frac{R^2}{2}\frac{\partial f_1}{\partial z} - Rf_3\right] + F_1$$

$$(w)_2 = \frac{Rf_1}{G_2} + \frac{1}{E_2}\frac{\partial}{\partial z}\left(\frac{R^3}{6}\frac{\partial f_1}{\partial z} - \frac{R^2}{2}f_3\right)$$

$$-R\frac{\partial F_1}{\partial z} + F_3 \qquad \left(-\frac{t_2}{2} \le R \le \frac{t_2}{2}\right) \tag{19}$$

where f_1, f_3, F_1, and F_3 are functions of z, which are to be determined from the continuity conditions given as

$$\overset{+}{u} = -\frac{t_2^2}{8E_2}\frac{\partial f_1}{\partial z} + F_1$$

$$\overset{-}{u} = \frac{t_2}{2E_2} f_3$$

$$\overset{+}{w} + \frac{t}{2}\frac{\partial \overset{-}{u}}{\partial z} = -\frac{t_2^2}{8E_2}\frac{\partial f_3}{\partial z} + F_3$$

$$\overset{-}{w} + \frac{t}{2}\frac{\partial \overset{+}{u}}{\partial z} = \frac{t_2}{2G_2} f_1 + \frac{t_2^3}{48E_2}\frac{\partial^2 f_1}{\partial z^2} - \frac{t_2}{2}\frac{\partial F_1}{\partial z} \tag{20}$$

It may be seen from Eq. (19) that the variation of radial displacement is of second degree while the longitudinal displacement has a cubic variation.

Results and Discussion

Numerical results have been obtained for the following cases:

1) $\eta_1 = \eta_3 = \eta = 0.01$, $\eta_2 = 0.02$, $\beta = 100\text{-}1000$

2) $\eta_1 = \eta_3 = \eta = 0.001\text{-}0.01$, $\eta_2 = 0.02$, $\beta = 500$

3) $\eta_1 = \eta_3 = \eta = 0.01$, $\eta_2 = 0.02\text{-}0.1$, $\beta = 500$

Typical elasticity results in respect of longitudinal stress (σ_z) are presented in Figs. 2a-c at one section for the parameters just given. It may be seen from these figures that the longitudinal stress varies linearly both in the outer and inner facings for the range of thicknesses of facings considered here. Furthermore, the magnitude of longitudinal stress is very small in the core for the range of core modulus considered here. Both these results indicate the applicability of sandwich shell theory for the range of parameters considered here.

A comparison of elasticity solution results with those of shell-core method and sandwich shell theories has been made for the cases already indicated. The results are presented in the form of percentage deviation in the maximum value of longitudinal stress (σ_z) and tangential stress (σ_θ) at the outer and inner interface and that of shear stress (τ_{rz}) at the midplane of the core, as well as outer and inner facings, and of longitudinal displacement (w) at the outer interface.

Variation of percentage errors in shell-core method and sandwich shell theories in respect of longitudinal stress (σ_z) at outer and inner interface, for the several parameters considered here, are presented in Figs. 3-5. Similarly, the variations of percentage errors in respect of tangential stress (σ_θ), shear stress (τ_{rz}), and longitudinal displacement (w) are presented in Figs. 6-14.

It may be seen from Figs. 3-5 that the maximum percentage deviation in shell-core method for the longitudinal stress at the outer interface, with β, η, η_2, varying in the ranges in-

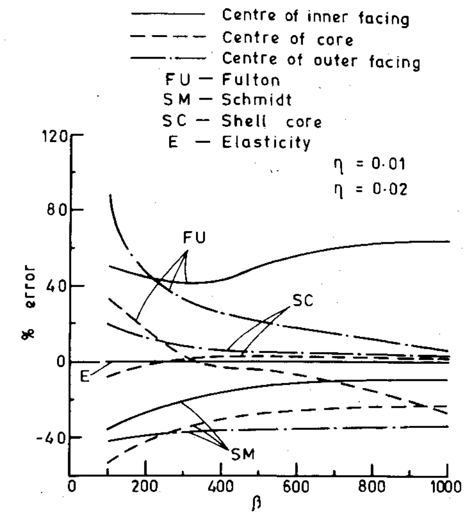

Fig. 9 Percentage error in τ_{rz} for different values of β at $z/r_2 = 0.06$.

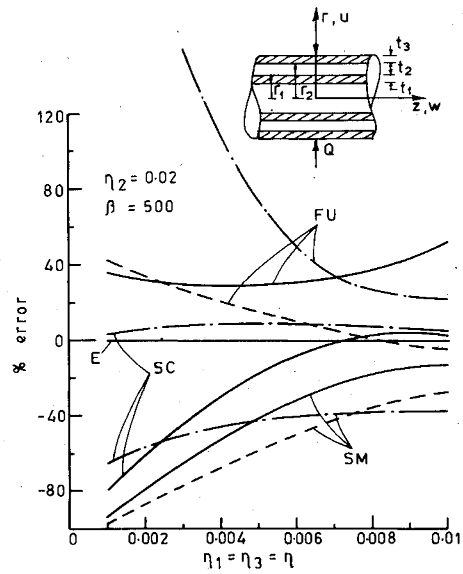

Fig. 10 Percentage error in τ_{rz} for different values of η at $z/r_2 = 0.06$.

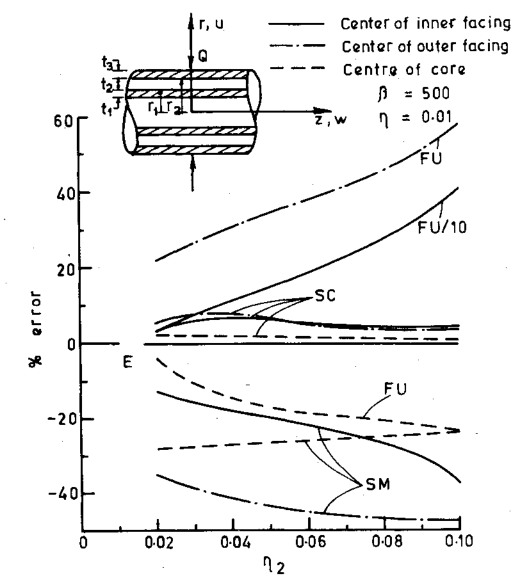

Fig. 11 Percentage error in τ_{rz} for different values of η_2 at $z/r_2 = 0.06$.

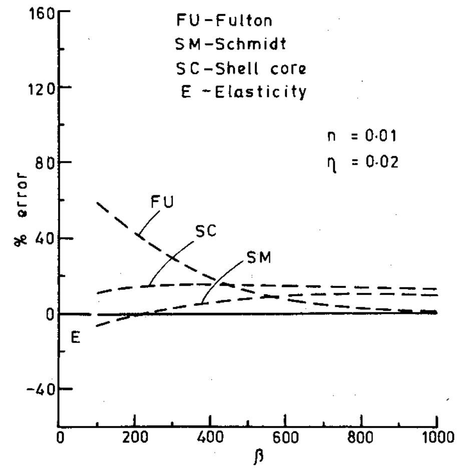

Fig. 12 Percentage error in longitudinal displacement (w) for different values of β at $z/r_2 = 0.06$ and outer interface.

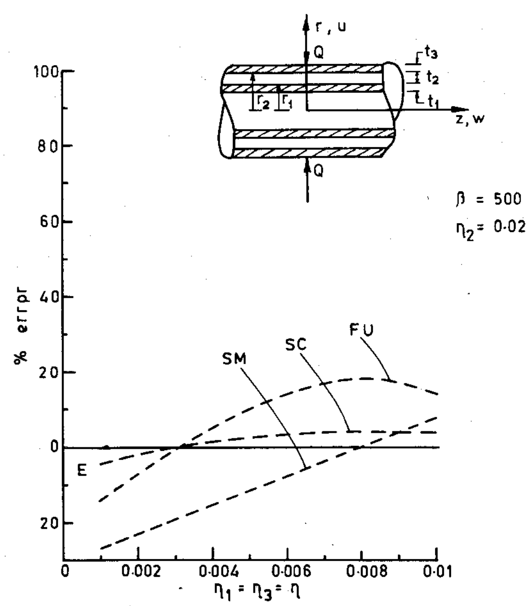

Fig. 13 Percentage error in longitudinal displacement (w) for different values of η at $z/r_2 = 0.06$ and outer interface.

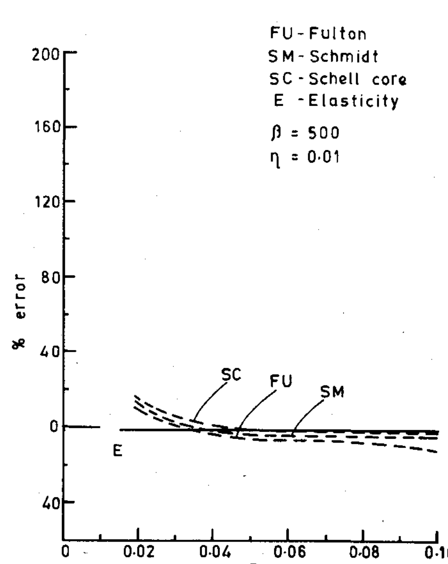

Fig. 14 Percentage error in longitudinal displacement (w) for different values of η_2 at $z/r_2 = 0.06$ and outer interface.

dicated earlier, is less than 10% while at the inner interface it is less than 15% only when $\beta > 500$. However, the maximum percentage error is less than 30% for all the parameters studied here. Further, for the cases considered here Schmidt's sandwich shell theory predicts the longitudinal stress better than Fulton's and the maximum error is $\pm 20\%$. Hence for the range of parameters considered here, it may be said that reasonably accurate results could be obtained by using either the shell-core method or Schmidt's theory. Sandwich shell theory due to Fulton appears to be reasonably accurate (except for inner interface) in the ranges $200 < \beta < 800$, $0.004 < \eta < 0.01$, $\eta_2 < 0.02$.

From Figs. 6-8 it may be seen that Schmidt's theory predicts σ_θ, both at inner and outer interface, within the error of $\pm 20\%$. Shell-core method predicts the outer interface stress better than Schmidt's, while at the inner interface the error is much larger than Schmidt's ($\pm 25\%$). It may be seen that if the facing thickness $\eta > 0.006$ shell core method predicts the shear stress (Figs. 9-11) at the midplane of the facings as well as the core extremely well and the error is less than 10%. Similarly, for $\eta > 0.006$ Schmidt's theory gives results which are about $\pm 40\%$ in error and Fulton's theory gives much larger error, sometimes exceeding 100%. From Figs. 12-14 it may be observed that the longitudinal displacement at the outer interface can be predicted very well using either shell-core solution or Schmidt's theory and the error is always less than 10%, while Fulton's method predicts the displacement with less than 20% error only for $\beta > 400$ and for all η and η_2. It was found that the longitudinal displacement at the inner interface was very small for the range of parameters considered here and, hence, the errors in respect to this are not presented here.

The large error at the inner interface in the Fulton's theory may be attributed to the assumption that radial displacement is constant over the thickness of core which is not corroborated by elasticity theory. This fact is further reinforced as in Schmidt's theory the transverse displacement is not constant and, hence, the results compare well with elasticity solution results.

Conclusions

For a sandwich cylindrical shell under axisymmetric load and for the range of shell parameters considered here it may be said that shell-core method and sandwich shell theory due to Schmidt gives reasonably accurate results and could be used. Sandwich shell theory as proposed by Fulton can give reliable results only in a small range of parameters considered here.

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